Series: AM411

SESSION – 2

Subject Code: MO

Roll No.: | | | | | | | | | | | | |

Candidates must write the Roll No on the title page of the Question Paper.

Mathematics | Level 4

Time allowed: 1.5 hours Maximum Marks: 150

Instructions for Candidates:

1. Please verify that this question paper contains **12** printed pages and Ensure that the paper contains a total of **50** questions.
2. Use of mobile phones, smartphones, ipads during examination is **STRICTLY PROHIBITED**.
3. On the OMR sheet, enter all required information carefully in the space provided **ONLY IN BLOCK CAPITALS** and by properly darkening the corresponding bubbles. Incomplete, incorrect, or careless entries may lead to disqualification.
4. Use only a **BLUE or BLACK BALL POINT PEN** for writing entries and filling bubbles. Pencils or other pens are not permitted.
5. **Marking Scheme:** Each question has four alternatives, only one of which is correct. Select the correct option and darken the respective bubble.
   – Each correct answer carries **3 marks**.
   – Each incorrect answer attracts a penalty of **1 mark**.
   – No marks are awarded for unanswered questions.
6. Use only the space provided in the question paper for rough work.
7. Candidates must remain in the examination hall until the exam is officially over. Early exit is not permitted.
8. Do not make stray marks, symbols, or extra writings on the answer sheet. Fill only the designated bubbles neatly.
9. Since answer sheets are machine evaluated, changes in entries are **strictly not allowed**.
10. Avoid scratching, overwriting, or erasing, as these may result in wrong evaluation.
11. **DO NOT write anything on the back side** of the answer sheet.

Multiple Choice Questions

Q1. The number of ways of a seven digit number with distinct digits of the form $a_1a_2a_3a_4a_5a_6a_7$, $(a_i \neq 0\ \forall i = 1, 2, \ldots, 7)$ be present in decimal system, such that

$$a_1 < a_2 < a_3 < a_4 > a_5 > a_6 > a_7$$

is

(A) 820

(B) 720

(C) 620

(D) 120

Q2. For a certain curve $y = f(x)$ satisfying

$$\frac{d^2y}{dx^2} = 6x - 4,$$

$f(x)$ has a local minimum value 5 when $x = 1$. The global maximum value of $f(x)$ if $0 \leq x \leq 2$, is

(A) 2

(B) 3

(C) 5

(D) 7

Q3. P is a variable point on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

whose foci are F_1 and F_2. Then maximum area of $\triangle PF_1F_2$ is

(A) $2a\sqrt{a^2 - b^2}$

(B) $2b\sqrt{a^2 - b^2}$

(C) $a\sqrt{2a^2 - 2b^2}$

(D) $b\sqrt{2a^2 - 2b^2}$

Q4. Distance of point $P(-2, 3, 4)$ from the line

$$\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$$

measured parallel to the plane $8x + 6y - 9z + 1 = 0$ is

(A) $\frac{\sqrt{301}}{2}$

(B) $\frac{\sqrt{401}}{2}$

(C) $\frac{\sqrt{307}}{2}$

(D) $\frac{\sqrt{207}}{2}$

Q5. The point $P(1, 1)$ is translated parallel to $2x = y$ in the first quadrant through a unit distance. The coordinates of the point P in new position are

(A) $\left(1 \pm \frac{2}{\sqrt{5}}, 1 \pm \frac{1}{\sqrt{5}}\right)$

(B) $\left(1 \pm \frac{1}{\sqrt{5}}, 1 \pm \frac{2}{\sqrt{5}}\right)$

(C) $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

(D) $\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

Q6. A function f from integers to integers is defined as

$$f(n) = \begin{cases} n+3, & n \text{ is odd} \\ \dfrac{n}{2}, & n \text{ is even} \end{cases}$$

If k is an odd integer and $f(f(f(k))) = 27$ then the sum of digits of k is

(A) 3

(B) 6

(C) 9

(D) 12

Q7. In a city, no two persons have identical set of teeth and there is no person without a tooth. Also no person has more than 32 teeth. If we disregard the shape and size of teeth and consider only the positioning of the teeth then the maximum population of the city will be

(A) 2^{32}
(B) $2^{32}+1$
(C) $2^{32}-1$
(D) $2^{32}-2$

Q8. The number of values of k, for which the system of equations

$$kx+(3k+2)y=4k$$

$$(3k-1)x+(9k+1)y=4(k+1)$$

has no solution, are

(A) 0
(B) 1
(C) 2
(D) 3

Q9. If $a \in [-20,0]$ then the probability that the graph of the function

$$y=16x^2+8ax+40x-7a-5$$

is strictly above the x-axis, is

(A) $\frac{14}{20}$
(B) $\frac{14}{21}$
(C) $\frac{13}{21}$
(D) $\frac{13}{20}$

Q10. If $1, z_1, z_2, \ldots, z_{n-1}$ are the n^{th} roots of unity then the value of

$$\frac{1}{3-z_1}+\frac{1}{3-z_2}+\cdots+\frac{1}{3-z_{n-1}}$$

is equal to

(A) $\frac{n\cdot 3^{n-1}}{3^n-1}+\frac{1}{2}$
(B) $\frac{n\cdot 3^{n-1}}{3^n-1}-1$
(C) $\frac{n\cdot 3^{n-1}}{3^n-1}+1$
(D) $\frac{n\cdot 3^{n-1}}{3^n-1}-\frac{1}{2}$

Q11. Let

$$f(x)=\int_2^x \frac{dt}{\sqrt{1+t^4}}$$

and g be the inverse of f. Then the value of $g'(0)$ is

(A) 1
(B) 17
(C) $\sqrt{17}$
(D) 0

Q12. The function

$$f(x)=[x]\cos\left(\pi\left(\frac{2x-1}{2}\right)\right),$$

(where $[\cdot]$ denotes the greatest integer function) is discontinuous

(A) for each real x
(B) for each integral point
(C) nowhere
(D) at each non-integral point

Q13. Let

$$f(x)=\lim_{n\to\infty}\frac{1}{\left(\frac{3}{\pi}\tan^{-1}2x\right)^{2n}}+5$$

then the set of values of x for which $f(x)=0$ is

(A) $|2x|>\sqrt{3}$
(B) $|2x|<\sqrt{3}$
(C) $|2x|\geq\sqrt{3}$
(D) $|2x|\leq\sqrt{3}$

Q14. Area bounded by the region

$$R \equiv \{(x,y) : y^2 \le x \le |y|\}$$

is

(A) $\frac{4}{3}$

(B) $\frac{3}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

Q15. The range of the function

$$f(x) = (1 + \sec^{-1} x)(1 + \cos^{-1} x)$$

is

(A) $(-\infty, \infty)$

(B) $(-\infty, 0] \cup [4, \infty)$

(C) $\{1, (1+\pi)^2\}$

(D) $\{0, (1+\pi^2)\}$

Q16. Consider the following equation in x and y:

$$(x-2y-1)^2+(4x+3y-4)^2+(x-2y-1)(4x+3y-4) = 0$$

How many solutions to (x, y) with x, y real, does the equation have

(A) zero

(B) exactly one

(C) exactly two

(D) more than two

Q17. If

$$f(n) = \int_0^{2015} \frac{e^x}{1+x^n}\,dx,$$

then $\lim_{n\to\infty} f(n)$ is equal to

(A) $e - 1$

(B) e

(C) 1

(D) $e^2 - 1$

Q18. Which of the following is NOT equivalent to $(p \wedge \sim q) \to r$

(A) $\sim (q \vee \sim p) \to r$

(B) $\sim r \to (\sim p \vee q)$

(C) $\sim \big((p \wedge (\sim q)) \wedge (\sim r)\big)$

(D) $\sim r \to (\sim p \wedge q)$

Q19. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set

(A) is increased by 2

(B) is decreased by 2

(C) is two times the original median

(D) remains the same as that of the original set

Q20. In a $\triangle ABC$, if $AB = 5$ cm, $BC = 13$ cm and $CA = 12$ cm, then the distance of vertex A from the side BC is (in cm)

(A) $\frac{25}{13}$

(B) $\frac{60}{13}$

(C) $\frac{65}{12}$

(D) $\frac{144}{13}$

Q21. Let $f(x)$ be a decreasing function defined on $(0, \infty)$. If $f(2a^2 + a + 1) < f(3a^2 - 4a + 1)$, then the range of a is

(A) $a > 1$ or $a < \frac{1}{3}$

(B) $a \in (0, 5)$

(C) $a > \frac{1}{3}$

(D) $a \in \left(0, \frac{1}{3}\right) \cup (1, 5)$

Q22. It is given that there are two sets of real numbers $A = \{a_1, a_2, \ldots, a_{100}\}$ and $B = \{b_1, b_2, \ldots, b_{50}\}$. Total number of onto functions f from A to B such that $f(a_1) \leq f(a_2) \leq f(a_3) \leq \ldots \leq f(a_{100})$ is/are

(A) ${}^{100}C_{50}$

(B) ${}^{99}C_{50}$

(C) ${}^{100}C_{49}$

(D) ${}^{99}C_{51}$

Q23. If $f(x) = 1 + \sum_{n=1}^{\infty} \dfrac{x^n (\log a)^n}{n!}$, then at $x = 0$, $f(x)$

(A) has no limit

(B) is discontinuous

(C) is continuous but not differentiable

(D) is differentiable

Q24. $\lim\limits_{n \to \infty} \cos(\pi \sqrt{n^2 + n})$, $n \in \mathbb{Z}$, is equal to

(A) 0

(B) 1

(C) -1

(D) does not exist

Q25. n/m means that n is a factor of m, then the relation '/' is

(A) reflexive and symmetric

(B) transitive and reflexive

(C) transitive and symmetric

(D) equivalence

Q26. In a triangle ABC, $\sum_{r=0}^{n} {}^nC_r a^r b^{n-r} \cos(rB - (n - r)A)$ is equal to (where a, b, c represent the sides and A, B, C represent the angles of $\triangle ABC$)

(A) a^n

(B) b^n

(C) c^n

(D) none of these

Q27. The radius of the circle touching the pair of lines $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$ and contained in the given circle is

(A) $\sqrt{10}$

(B) 3

(C) $2\sqrt{2}$

(D) $\sqrt{7}$

Q28. A parabola having directrix $x + y + 2 = 0$ touches a line $2x + y - 5 = 0$ at $(2, 1)$. Then the semi-latus rectum of the parabola is

(A) $\dfrac{8}{\sqrt{2}}$

(B) $\dfrac{9}{\sqrt{2}}$

(C) $\dfrac{10}{\sqrt{2}}$

(D) $\dfrac{11}{\sqrt{2}}$

Q29. Number of points from which pair of perpendicular tangents can be drawn to hyperbola $x^2 \csc^2 \alpha - y^2 \sec^2 \alpha = 1$, is $\alpha \in \left(0, \dfrac{\pi}{4}\right)$

(A) 0

(B) 1

(C) 2

(D) infinite

Q30. Let $f(x) = \sin x + \cos x + \tan x + \sin^{-1} x + \tan^{-1} x + \cos^{-1} x$. If M and m are maximum and minimum values of $f(x)$, then its sum is

(A) $\pi + 2\cos 1$

(B) $\pi + 2\sin 1$

(C) $\frac{\pi}{2} + 2\tan 1$

(D) $\pi + \tan 1 + \sin 1$

Q31. In a right angled triangle, the smallest angle is θ. A triangle which is taking the reciprocal of sides is also a right angled triangle, then $\sin\theta$ is equal to

(A) $\frac{\sqrt{5}+1}{4}$

(B) $\frac{\sqrt{5}-1}{2}$

(C) $\frac{\sqrt{5}-1}{4}$

(D) $\frac{\sqrt{2}-1}{2}$

Q32. $\lim\limits_{n\to\infty} \sum\limits_{r=1}^{n} \cot^{-1}\left(\frac{r^3-r+1}{r}\right)$ is equal to

(A) 0

(B) π

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

Q33. Let $f(\theta) = \frac{1}{1+(\cot\theta)^x}$, then the value of $2\sum\limits_{\theta=1^\circ}^{89^\circ} f(\theta)$ is

(A) 44

(B) 89

(C) 65

(D) 178

Q34. For any three vectors $\vec{a}, \vec{b}, \vec{c}$ (non-zero), $(\vec{a}-\vec{b})\cdot[(\vec{b}-\vec{c})\times(\vec{c}-\vec{a})]$ is

(A) $2|\vec{c}|$

(B) $3|\vec{b}-\vec{c}|$

(C) $3|\vec{b}-\vec{c}+\vec{a}|$

(D) zero

Q35. The projection of the line segment $P(1,2,3)$ and $Q(-1,4,2)$ on the line having direction ratios $2,3,-6$ is

(A) $\frac{7}{8}$

(B) 6

(C) $\frac{7}{8}$

(D) $\frac{9}{2}$

Q36. The number of distinct terms in the expansion of $(2x+3+\frac{1}{5x})^n$, $n \in \mathbb{N}$ is

(A) $3n$

(B) $3n+1$

(C) $2n$

(D) $2n+1$

Q37. In a G.P. of positive terms, product of first four terms is 4 and second term is the reciprocal of fourth term. Then sum of infinite G.P. is

(A) divisible by 3

(B) divisible by 5

(C) divisible by 2

(D) an irrational number

Q38. If the system of equations

$$x-2y+z=a; \quad 2x+y-2z=b; \quad x+3y-3z=c$$

has at least one solution, then

(A) $a+b+c=0$

(B) $a-b+c=0$

(C) $-a+b+c=0$

(D) $a+b-c=0$

Q39. If $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$, then the modulus and argument of $(1+\cos 2\alpha)+i\sin 2\alpha$ are respectively

(A) $2\cos\alpha, \alpha$

(B) $-2\cos\alpha, \alpha$

(C) $-2\cos\alpha, \alpha-\pi$

(D) None of these

Q40. The statement $p \to (\sim q)$ is equivalent to

(A) $q \to p$

(B) $p \wedge \sim q$

(C) $\sim q \vee \sim p$

(D) $\sim q \to p$

Q41. Consider the ellipse

$$\frac{x^2}{4}+\frac{y^2}{3}=1.$$

Let $H(\alpha, 0)$, $0<\alpha<2$, be a point. A straight line drawn through H parallel to the y-axis crosses the ellipse and its auxiliary circle at points E and F respectively, in the first quadrant. The tangent to the ellipse at point E intersects the positive x-axis at point G. Suppose the straight line joining F and the origin makes an angle ϕ with the positive x-axis.

List–I	**List–II**
(I) If $\phi=\frac{\pi}{4}$, then area of $\triangle FGH$ is	(P) $\frac{(\sqrt{3}-1)^4}{8}$
(II) If $\phi=\frac{\pi}{3}$, then area of $\triangle FGH$ is	(Q) 1
(III) If $\phi=\frac{\pi}{6}$, then area of $\triangle FGH$ is	(R) $\frac{3}{4}$
(IV) If $\phi=\frac{\pi}{12}$, then area of $\triangle FGH$ is	(S) $\frac{1}{2\sqrt{3}}$
	(T) $\frac{3\sqrt{3}}{2}$

The correct option is:

(A) (I) → (R); (II) → (S); (III) → (Q); (IV) → (P)

(B) (I) → (R); (II) → (T); (III) → (S); (IV) → (P)

(C) (I) → (Q); (II) → (T); (III) → (S); (IV) → (P)

(D) (I) → (Q); (II) → (S); (III) → (Q); (IV) → (P)

Q42. Let f be the function defined by $f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n}$ for all x such that $-1<x<1$. Then $f'(x)=$

(A) $\frac{1}{1-x}$

(B) $\frac{x}{1-x}$

(C) $\frac{1}{1+x}$

(D) $\frac{x}{1+x}$

(E) 0

Q43. If z is a complex variable and $\bar{z}$ denotes the complex conjugate of z, what is

$$\lim_{z\to 0}\frac{(\bar{z})^2}{z^2}?$$

(A) 0

(B) 1

(C) i

(D) ∞

Q44. Let g be the function defined by $g(x)=e^{2x+1}$ for all real x. Then

$$\lim_{x\to 0}\frac{g(g(x))-g(e)}{x}=$$

(A) $4e^2$

(B) e^{2e+1}

(C) $2e^{2e+1}$

(D) $4e^{2e+2}$

Q45. What is the value of

$$\int_{-\pi/4}^{\pi/4}\left(\cos t+\sqrt{1+t^2}\sin^3 t\cos^3 t\right)\,dt?$$

(A) $\sqrt{2}$

(B) $\sqrt{2}-1$

(C) $\dfrac{\sqrt{2}}{2}$

(D) $\dfrac{\sqrt{2}-1}{2}$

Q46. What is the volume of the solid in xyz–space bounded by the surfaces $y = x^2$, $y = 2 - x^2$, $z = 0$, and $z = y + 3$?

(A) $\dfrac{8}{3}$

(B) $\dfrac{16}{3}$

(C) $\dfrac{104}{105}$

(D) $\dfrac{208}{105}$

Q47. Which of the following statements about the real matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

is **FALSE**?

(A) A is invertible.

(B) If $\mathbf{x} \in \mathbb{R}^5$ and $A\mathbf{x} = \mathbf{x}$, then $\mathbf{x} = 0$.

(C) The last row of A^2 is $(0\ 0\ 0\ 0\ 25)$.

(D) A can be transformed into the 5×5 identity matrix by a sequence of elementary row operations.

Q48. In xyz–space, what are the coordinates of the point on the plane $2x + y + 3z = 3$ that is closest to the origin?

(A) $\left(\dfrac{3}{7}, \dfrac{3}{14}, \dfrac{9}{14}\right)$

(B) $\left(\dfrac{7}{15}, \dfrac{8}{15}, \dfrac{1}{15}\right)$

(C) $\left(\dfrac{5}{6}, \dfrac{1}{3}, \dfrac{1}{3}\right)$

(D) $(1, 1, \frac{1}{3})$

Q49. Let S be the set of all functions $f : \mathbb{R} \to \mathbb{R}$. Consider the two binary operations $+$ and $\circ$ on S defined as pointwise addition and composition of functions, as follows:

$$(f+g)(x) = f(x)+g(x), \quad (f\circ g)(x) = f(g(x)).$$

Which of the following statements are true?

I. $\circ$ is commutative.

II. $+$ and $\circ$ satisfy the left distributive law $(f \circ (g + h)) = (f \circ g) + (f \circ h)$.

III. $+$ and $\circ$ satisfy the right distributive law $((g + h) \circ f) = (g \circ f) + (h \circ f)$.

(A) II only

(B) III only

(C) II and III only

(D) I, II, and III

Q50. A tank initially contains a salt solution of 3 grams of salt dissolved in 100 liters of water. A salt solution containing 0.02 grams of salt per liter of water is sprayed into the tank at a rate of 4 liters per minute. The sprayed solution is continually mixed with the salt solution in the tank, and the mixture flows out of the tank at a rate of 4 liters per minute. If the mixing is instantaneous, how many grams of salt are in the tank after 100 minutes have elapsed?

(A) $2 - e^{-2}$

(B) $2 + e^{-2}$

(C) $2 - e^{-4}$

(D) $2 + e^{-4}$

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Space for Rough Work

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2) Challenge Your Parent / Friend

Pick **any 3 questions** from this paper and ask your parent/sibling/friend to solve them.

Q. No.	Solved? (Y/N)	Time Taken	Who explained better?

3) Teacher's Corner

Teachers can reuse this paper for continued learning:

- Select 5 questions for a **weekly quiz**.
- Discuss the **most missed** question in class.
- Post one **"Question of the Week"** on the noticeboard.

4) Next Steps for Students

- File this paper in your **ScienceX Portfolio**. Reattempt it in **2 weeks** and compare scores.
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